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Soil Stratification and Ponded Flow into Subsurface Drains

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SOIL STRATIFICATION AND PONDED FLOW INTO SUBSURFACE DRAINS

WILLIAM BURKE AND GEORGE S. TAYLOR[1]

INTRODUCTION

Significant advances have been made in drainage theory during the past two decades, particularly insofar as these theories involve the seepage of water through soils. Much of the theory concerns steady-state ponded flow. Kirkham (5) pointed out that the theory of ponded flow is of concern for two reasons: First, because the removal of ponded water by underdrains is in itself of interest; second, because the theory illustrates hydraulic principles in other seepage problems. Furthermore, the theoretical rate of movement of ponded water represents an upper limit of flow for the conduction of water through soil to a drainage facility.

The appropriate equation for two-dimensional steady-state, laminar flow in a saturated porous medium is the Laplace equation:

$$\frac{\partial}{\partial x}\left(K\frac{\partial \Phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial \Phi}{\partial y}\right) = 0 \qquad \text{Eq. (1)}$$

where K is the hydraulic conductivity of the porous medium and Φ is the hydraulic head. The solution of a particular drainage problem thus reduces to the solution of Laplace's equation for the boundary conditions specific to that problem. Kirkham has developed an exact analytical solution for the case of ponded flow into drain tubes in a uniform soil overlying an impervious layer (3) and also for seepage into drain tubes in two-layered soils (4). These solutions have been of great value in showing the influence of drain size, depth, and spacing, and of soil hydraulic conductivity on flow in both uniform and strati-

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fied soil. The analytic solutions are generally in the form of infinite series in logarithmic hyperbolic functions, and they apply only to somewhat regular boundary conditions. In addition to analytical solution of drainage problems, various types of analogs (2, 6, 7, 9) have also been useful in solving equation (1) for various problems in drainage.

In the report which follows, a study was made of the effect of soil stratification on ponded flow into underground drains. This was done by solving equation [1] for the appropriate boundary conditions imposed by the flow problem. A numerical analysis procedure was used and the calculations were made by an electronic computer. The cases chosen for analysis were not meant to represent actual soils. Rather, they were chosen to evaluate the flow characteristics of soils having two or three layers which differ significantly in their hydraulic conductivity.

EXPERIMENTAL

The drainage case analyzed was as follows: Drain tubes of radius r are buried at a depth d in saturated soil and are running full with no back pressure. The drains are essentially horizontal and their walls are infinitely permeable. The drains are considered to be of infinite length so that flow into the drains is of two-dimensional character. The soil is layered but isotropic with respect to its hydraulic conductivity K. The ground surface is covered with a continuously maintained thin film of water. The soil is considered to consist of two or three layers, each layer having variable thickness and hydraulic conductivity. A steady-state flow condition exists so that inflow across the ground surface is equal to that entering the drain.

The procedure followed was to solve equation [1] in terms of the hydraulic head Φ for the case described above. This was done by utilizing a numerical analyses procedure, and this procedure was programmed on an electronic computer. Details of the procedure are given in a previous report (8), and only a few general remarks will be made here. The solution of equation [1] yielded values of Φ at various points of a grid laid over the flow region. (i.e. see Figure 2 of reference number 8). By interpolating these values between grid points, lines of equal potential were obtained. The water entering the ground surface per unit length of drain (i.e. Q) was determined by summing up the flux between grid points at the ground surface (8, p. 551). The intersection of the streamlines with the ground surface was given by the horizontal distances at which 20%, 40%, etc., of Q entered the ground surface. Streamlines were drawn orthogonal to the equipotentials.

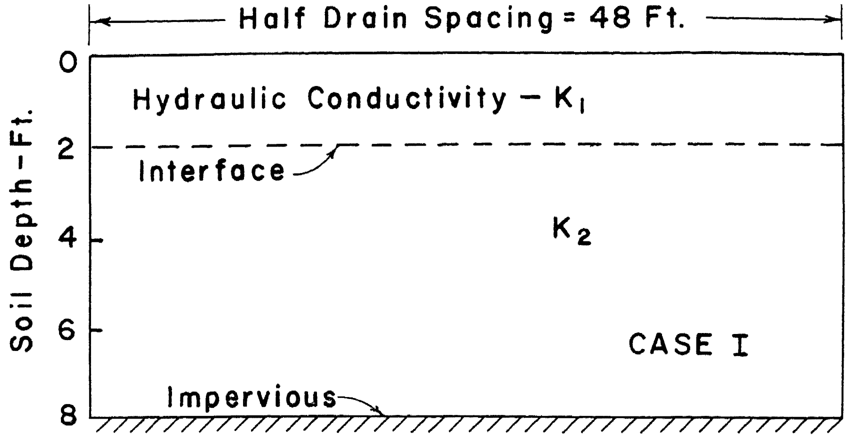

Fig. 1.—Schematic representation of the soil studied in Case I.

RESULTS AND DISCUSSION

Case I

The soil represented here is two-layered as shown in Figure 1. A shallow layer of soil hydraulic conductivity K_1 overlies a thicker one of conductivity K_2. The magnitude of K_1 is variable, but in the discussion which follows it is assumed to be a constant value of 1 foot per day. The magnitude of K_2 never exceeds K_1, and each layer is isotropic with respect to its conductivity. Drain lines of 4-inch diameter are considered to be installed at a spacing of 96 feet, while the drain depth is considered to vary from one to eight feet by one-foot increments. The drain line spacing is sufficiently large so that flow into the drains is essentially that for infinite spacing.

The results of these analyses[2] are shown in Figure 2 as a function of drain depth. The numerical results are given in Table 1, and some of the flow nets are shown in Figures 7, 8, and 9 of the Appendix. The drain flow rates Q are not given directly but are expressed in terms of the quantity Q/K_1 for purposes of generality. Thus the values on the abscissa apply to all values of K_1 and K_2 as long as their ratio K_1/K_2 is the same as shown therein. For convenience, the quantity Q/K_1 will hereafter be called "flow rate" in this report. The solid line curve at the extreme right of Figure 2 gives the flow rate as determined by the analytic solution of Kirkam (3), while the computer-derived values are represented by the circled points. In the other curves, the solid lines represent a visual fit of the computer results. The deviation of the

[2]A portion of the data illustrated in Figure 2 was reported in an earlier article (8).

computer results from Kirkham's analytic solution was always less than 5 percent. The analytic solution normally yielded higher values for flow rates.

The effect of drain depth on flow rate in an unlayered soil is shown at the right of Figure 2. Flow rates increase with drain depths until the drain approaches the impervious layer. The drain depth at which the flow rate shows a sharp decline is approximately 7 feet. At this drain location, the distance between the drain and the impervious layer is about 10% of the total soil depth. When the drain is at the 8-foot depth, only the upper half of the drain receives water since the lower half is embedded in the impervious layer. The lower flow rate for this drain depth is only partially due to the "half drain" effect since flow rates are also reduced at the 7-foot location. The major reason for low flow rates at and near the impervious floor appears to be the restricted flow region adjacent to the drain.

When the drain is located in the upper soil layer, flow rates are higher at the 2-foot depth than at one foot. The maximum value of the flow rate in this layer is probably reached at some drain depth between one and two feet, particularly for high ratios of K_1/K_2. This drain depth is not ascertainable from the data since analyses were not obtained for drain depths within this interval. When the conductivity of the lower layer is equal to or less than one-tenth K_1, flow rates from drains placed in the upper layer are essentially unaffected by the conductivity of the lower one. In other words the magnitude of K_2 is so small compared to K_1 that the interface acts as an impervious floor. When K_2 is 1/10, 1/5, 1/3, and 1/2 that of K_1, the flow rates at the

TABLE 1.—Values of Q/K_1 for Various Drain Depths in the Two-Layered Soils Illustrated by Case 1. The Drain Diameter Is 4 Inches, and the Drain Spacing Is 96 Feet.

Drain Depth (ft.)	Ratio K_1/K_2					
	1	2	3	10	50	100
1	1.10	1.04	1.02	1.00	0.98	0.98
2	1.76	1.39	1.27	1.10	1.04	1.03
3	2.29	1.36	0.97	0.32	0.07	0.03
4	3.00	1.57	1.09	0.36	0.07	0.04
5	3.27	1.50	1.22	0.37	0.08	0.04
6	3.84	1.94	1.32	0.43	0.09	0.04
7	3.69	1.92	1.29	0.39	0.08	0.04
8	2.90	1.51	1.04	0 34	0.07	0.04

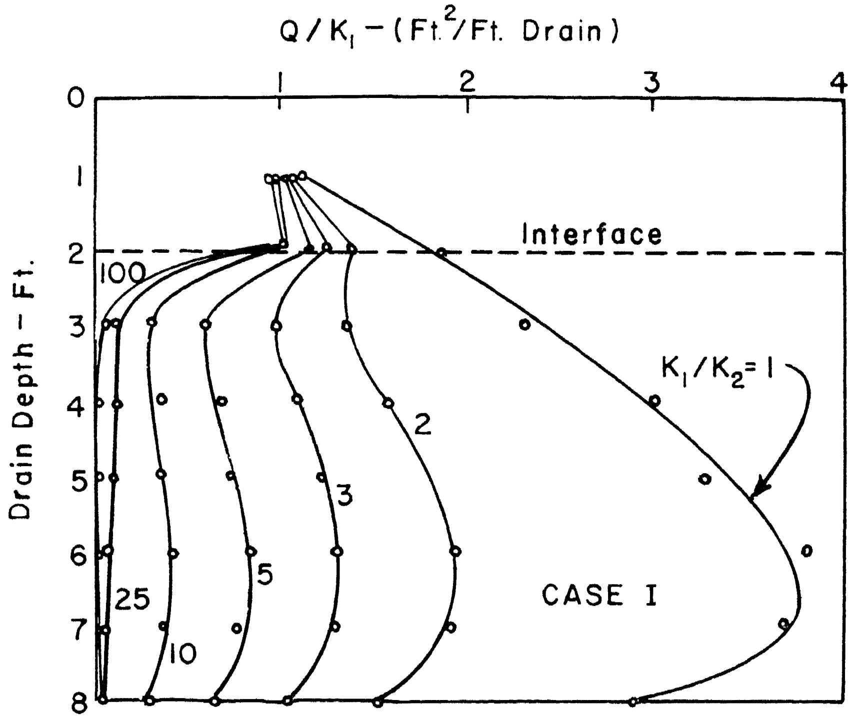

Fig. 2.—Values of Q/K_1 for various drain depths in the two layered soil illustrated by Case I. Q is the flow per unit time per unit length of the drain, and K_1 is the hydraulic conductivity of the upper soil layer.

two-foot drain depth are, respectively, 63, 68, 72, and 80% that from the unlayered soil of conductivity K_1.

For K_1/K_2 ratios equal to or greater than 2, a reduction in flow results from placing the drain in the top portion of the lower, less permeable layer. Some compensation in flow rates is obtained by placing the drain at greater depths in this layer. For drains in the lower layer, maximum flow rates occur when the drain is 1.0 foot above the impermeable layer. For conductivity ratios greater than 3, maximum flow for all drain depths occurs when the drain is at the layer interface. For ratios less than 3, higher flow rates can be achieved by placing the drain sufficiently deep in the lower layer.

Since the hydraulic carrying capacity of drains usually exceeds the rate of water entry, one might conclude the following from the information presented in Figure 2: For unlayered soils, greatest flow rates occur if the drains are just above an impervious layer. There is

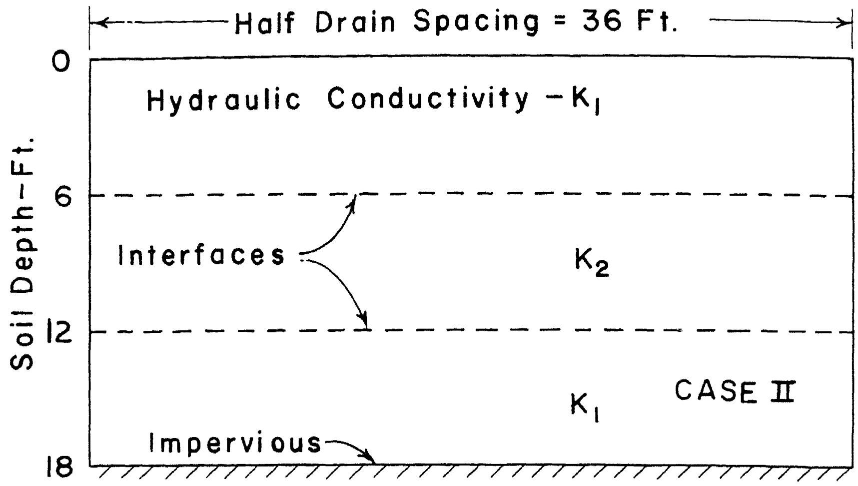

Fig. 3.—Schematic representation of the soil studied in Case II.

every reason to believe that water table drawdown will also be more rapid if the drains are installed deep. Thus for most rapid drainage, it would appear that drains should be installed as deeply as practical in such soils but above an impervious layer.

Greater caution must be followed in similar interpretations for layered soils. For example, one must weigh the greater conductivity in the top layer against the greater head resulting from deeper drain installations. If the layer interface occurs from 2 to 4 feet, it would appear that for conductivity ratios less than 4 or 5 that most rapid drawdown would result from installing the drain in the upper part of the bottom layer. For ratios greater than 10, drains located at the layer interface would probably yield the most rapid drawdown. On the other hand, if under field conditions the backfill soil region has a conductivity as great as K_1, deep placement of drains should always yield fastest (or equal) drawdown in layered soil. A more thorough analysis of drawdown for these situations must await analyses of the falling water table case in drainage.

Interpretation of the results shown in Figure 2 can also be extended to the situation where all physical dimensions of Case I are scaled upward or downward (2). For example, one would obtain the same curves for the situation where the interface occurs at 4 feet, the impervious layers is at 16 feet, and the drain diameter and spacing are also doubled. The scale on the ordinate and abscissa of Figure 1 must also be doubled in this example; otherwise, the curves remain unchanged.

In other words, one can interpret the information given in Figure 2 for a layer interface at 4 rather than 2 feet.

The nature of flow in Case I is illustrated in Figures 7, 8, and 9. For the unlayered soil (Figure 7), equipotential lines are more concentrated near the drain when the latter is either near the ground surface or the impermeable layer. This is a result of the very small horizontal gradients in the former and of the restricted flow region near the drain in the latter. The effect of increasing the drain depth is to displace the streamlines away from the drain. In general, the effect of decreasing the permeability of the lower layer is to displace the streamlines towards the drain.

Drain depth has a much stronger effect on streamline displacement than has the magnitude of the ratio K_1/K_2. The positions of the equipotential lines relative to the drains is not greatly affected either by drain depth or by the conductivity ratio.

Case II

This soil is assumed to consist of three layers of equal thickness and is also underlain by an impervious stratum as shown in Figure 3. The objective is to determine drain flow rates for this soil when the middle layer has either a lower or higher[3] hydraulic conductivity than the other two. The upper and lower layers have the same hydraulic conductivity K_1. While K_1 may be of any magnitude, for our discussion it is assumed to have a fixed value of 1 foot per day. The magnitude of K_2 ranges from fifty times greater to one-fiftieth that of K_1. As with Case I, the drain diameter is 4 inches, although the spacing between drain lines is 72 instead of 96 feet. Drain flow rates are determined for drain depths of 4, 6, 9, and 14 feet. These results are shown graphically in figure 4. Numerical values are given in Table 2, and some of the flow nets are shown in Figures 10, 11, and 12 of the Appendix.

First, one should note that the series of curves on the left side of Figure 4 has a different horizontal scale from those on the right. The curves at the upper left and upper right represent flow rates in a homogeneous soil of conductivity K_1, and the two curves are identical except for horizontal scale. None of the curves represents data for

[3]It has been shown by Day and Luthin that negative hydrostatic pressure may arise in a stratum which is overlain by one having a lower hydraulic conductivity. (See Soil Sci. Soc. Amer. Proc., 17:87-91, 1953). In theory, the occurrence of negative pressures offers no problem to the solutions at hand. In practice, the reduced pressure may cause dissolved gases to form in some of the pore spaces and thus reduce the hydraulic conductivity. In the analyses which follow, it is assumed this does not occur.

TABLE 2.—Values of Q/K_1 for Various Drain Depths in the Three-Layered Soil Illustrated by Case II. The Drain Diameter Is 4 Inches, and the Drain Spacing Is 72 Feet.

Drain Depth (ft.)	Ratio K_1/K_2 50	10	5	1	1/5	1/10	1/50
4	2.76	2.81	2.85	3.07	3 47	3.69	4.75
6	2.45	2.62	2.82	4.01	9.09	13.19	25.54
9	0.25	0.82	1.45	5.41	15 49	22.39	39.81
14	1.21	3.33	4.35	7.00	9.94	11.01	15.10

drain depths very close to the impervious floor. Consequently, the curves do not show the characteristic reduction in flow rates at greater depths as shown in Figure 2.

The conductivity of the middle layer has a pronounced effect on drain flow rates, particularly if the drain is either in or below it. Undoubtedy, this is a result of the high percentage of head loss which is dissipated near the drain. This explains the large influence of the layer in which the drain is located.

When the drain is located in the center layer, flow rates are altered relatively more if K_2 is reduced in magnitude than if K_2 is increased in the same proportion. Consider, for example, the situation when K_2 is first reduced from K_1 to $K_1/10$, and then is increased from K_1 to 10K. The drain flow rates are one-seventh that of the unlayered soil in the first example but only four times as large as the unlayered soil in the second. The explanation is that 70% of the hydraulic head loss occurs near the drain when K_2 is of low magnitude while only 30% occurs when K_2 is high (see Figures 11 and 12 of the Appendix). In the first case flow rates are dominated by the low conductivity in the center layer; whereas, in the latter, they are not. These results agree in general with those obtained by Luthin (6) with an electrical resistance network.

Flow nets for some situations of Case II are shown in Figures 10, 11, and 12. Compared to the unlayered soil of Case I, the streamlines shown in Figure 10 are displaced farther away from the drain, the equipotentials are less concentrated near the drain, and the drain flow rate is higher. These differences are primarily a result of the greater depth to an impermeable floor in Case II.

In Figure 11, the hydraulic conductivity in the middle layer is one-tenth that of the other two. When the drain is in the upper layer, 70% of the drop in potential occurs within a horizontal distance of less

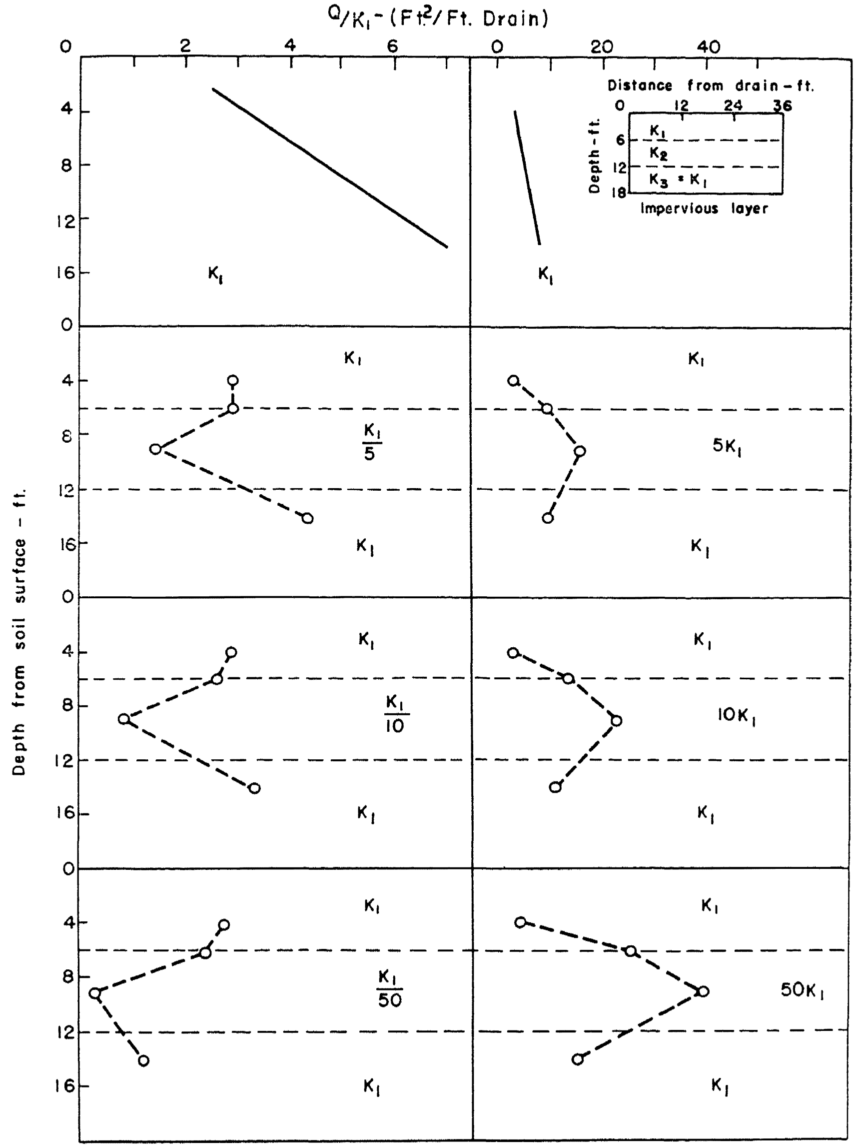

Fig. 4.—Values of Q/K_1 for various drain depths in the soil illustrated by Case II.

than 4 feet from the drain, and 70% of flow occurs within a horizontal distance of 6 feet. Placing the drain in the middle layer causes most of the loss in potential to occur still closer to the drain, but it displaces the streamlines outwards and distributes them more uniformly. Only 5 percent loss in potential occurs in the upper layer. Placing the drain in the bottom layer gives a vertical direction and a more uniform spatial distribution to streamlines in the two upper layers. Equipotential lines are essentially horizontal in these layers. In the bottom layer equipotentials are shifted towards the vertical. Fifty percent of the drop in potential occurs in the middle layer directly over the drain, while in the bottom layer this drop is spread over a horizontal distance of 26 feet.

In the soil to which the flow nets in Figure 12 apply, the middle layer has a hydraulic conductivity ten times greater than that in the top and bottom layers. This is the reverse of the conditions shown in Figure 11. When the drain is in the upper layer, most of the drop in potential occurs in the region of the drain. This is also true for the situation shown in Figure 11. However, the 70 percent streamline is about 16 feet from the drain as compared with 6 feet for the conditions shown in Figure 11. The distribution of streamlines in the upper layer varies little, irrespective of the layer in which the drain is located. This is due to the high hydraulic conductivity of the middle layer which causes water to move vertically downwards through the top layer.

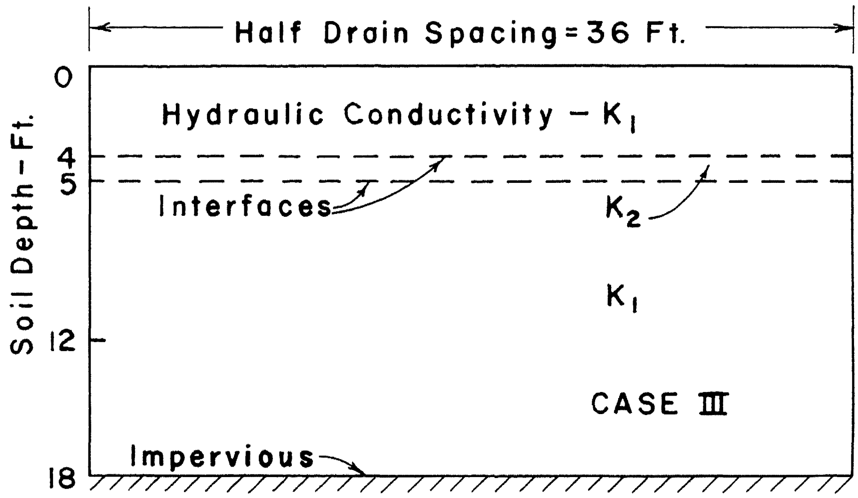

Fig. 5—Schematic representation of the soil studied in Case III.

TABLE 3.—Values of Q/K_1 for Various Drain Depths in the Three-Layered Soils Represented by Case III. The Drain Diameter Is 4 Inches, and the Drain Spacing Is 72 Feet.

Drain Depth (ft.)	Ratio K_1/K_2 1	5	10	50	100
2	1.79	1.74	1.72	1.68	1.68
4	3.07	2.20	2.02	1.84	1.81
5	3.51	2.19	1.82	1.11	0.82
8	4.98	4.35	3.86	2.31	1.62
9	5.41	4.80	4.30	2.62	1.84
11	6.18	5.61	5.09	3.18	2.24
14	7.00	6.51	6.02	3.95	2.83
17	7.34	6.90	6.47	4.49	3.33
18	5.89	5.61	5.34	3.99	3.09

When the drain is in the middle layer, there is a drop of about 70 percent directly over the drain. In the middle layer, the 70 percent equipotential is almost vertical and about 25 feet from the drain. For the drain in the bottom layer, most of the loss in potential occurs in the vicinity of the drain. The 70 percent equipotential line is about 6 feet from the drain at its greatest horizontal distance, and there is a potential drop of approximately 15 percent in each of the two upper layers directly over the drain.

A comparison of Figures 11 and 12 shows the following characteristics: When the drain is in the layer of lowest hydraulic conductivity, most of the loss in potential occurs near it. Equipotential lines are most evenly distributed when the drain is in the more permeable layer, except when this is a surface layer overlying one of restricted hydraulic conductivity (Figure 11, top section). In all cases, equipotential lines in the upper layer are essentially horizontal in direction; thus, the streamlines are almost vertical. The opposite is true of the bottom layer. When the middle layer has a high hydraulic conductivity relative to the others, equipotential lines in this layer have a general vertical direction and streamlines are essentially horizontal. The opposite holds true when the hydraulic conductivity of the middle layer is low.

Case III

The soil represented in this case is illustrated in Figure 5. This soil also has three layers, but the middle layer is thin, relatively near the surface, and has a lower conductivity than the other two layers. The objective is to evaluate the effect of such a layer on drain flow rates when the drain is located at various depths in the soil profile. As with

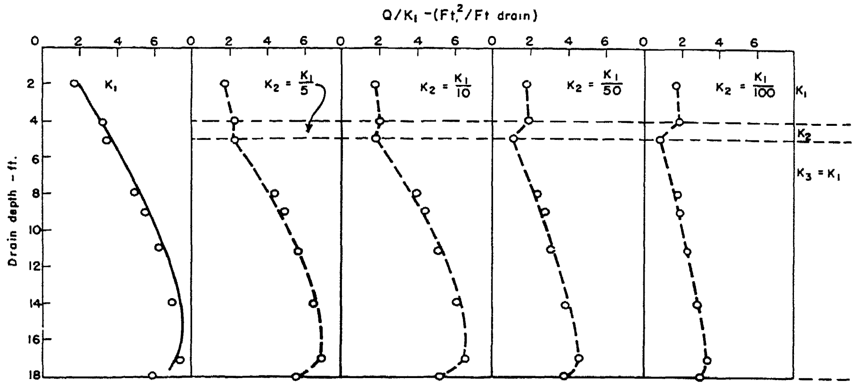

Fig. 6.—Values of Q/K_1 for various drain depths in the soil illustrated by Case III.

Cases I and II, K_1 is assigned a magnitude of 1 foot per day. The results of these analyses are shown in Figure 6. Numerical values are shown in Table 3, and some of the flow nets are shown in Figure 13.

Drain flow rates are shown for a homogeneous soil at the extreme left of Figure 6 and for various magnitudes of K_2 in the remainder of the figure. It is evident that two factors determine flow rate. These are the ratio K_1 to K_2 and the drain depth. For K_2 equal to $K_1/5$ or $K_1/10$, minimum flow occurs when the drain is at the two-foot depth in the upper layer. When K_2 is $K_1/50$ and $K_1/100$, flow rates are essentially equal when the drain is at the lower interface or at two feet. Flow in the layered soils is always greater for drains at the upper than at the lower interface. When the drain is in the lowest layer, flow rates increase with depth to a point about 1.0 foot from the impermeable layer. By placing drains sufficiently deep, flow rates can be obtained which are greater than those for drains in the upper layer.

The required depth depends on the value of K_2. For example when K_2 is equal to $K_1/5$, a drain placed slightly below the interface is as effective as one placed at the upper interface. If K_2 is $K_1/100$, the drain must be at almost 9 feet (that is, 4 feet below the interface) to achieve the same flow rate as for a drain at the upper interface.

The flow nets shown in Figure 13 represent a soil having a thin layer of low permeability near the surface (Case III). Flow nets in general are similar to those in the three-layered soil described in Figure 11.

SUMMARY AND CONCLUSIONS

An electronic computer was used in the study of steady state problems of ponded flow in homogeneous and in layered soils. The method of solution was by numerical analysis. Use of the computer permitted the rapid study of 144 separate drainage situations. The drainage characteristics of two- and three-layered soils were studied, together with those of homogeneous soils of similar dimensions.

The presence of layers of different hydraulic conductivities was shown to have a pronounced effect on drain flow rates and also on potential and streamline distribution. In a two-layered soil in which the upper layer had a higher hydraulic conductivity than the lower, flow rates were generally greater for drains in the upper layer or at the layer interface than for drains in the lower layer. The exceptions occurred when the lower layer had a conductivity at least one-third that of the upper one.

In a three-layered soil in which each layer was of equal thickness, maximum flow rate was achieved by placing the drain in the middle layer when this layer had a higher hydraulic conductivity than the other two. When the middle layer had a lower hydraulic conductivity, flow rates were reduced considerably if the drain was placed in this layer. For maximum flow rates in a soil of the latter type, drains should be placed in the upper layer. These results are in agreement with those obtained by Luthin using an electrical resistance network.

In a three-layered soil, the middle layer of which was thin and had a low hydraulic conductivity, flow patterns were similar to those in a corresponding soil having layers of equal thickness. For drains in the upper layer, maximum flow rate was achieved when the drain was at the interface. Higher flow rates can be obtained by placing the drains in the bottom layer, but as the ratio of K of the upper layer to K of the middle one increases, so does the depth to which drains must be placed. To achieve maximum flow rates for drains in the bottom layer, the drains should be placed as deeply as possible but not on the impermeable layer.

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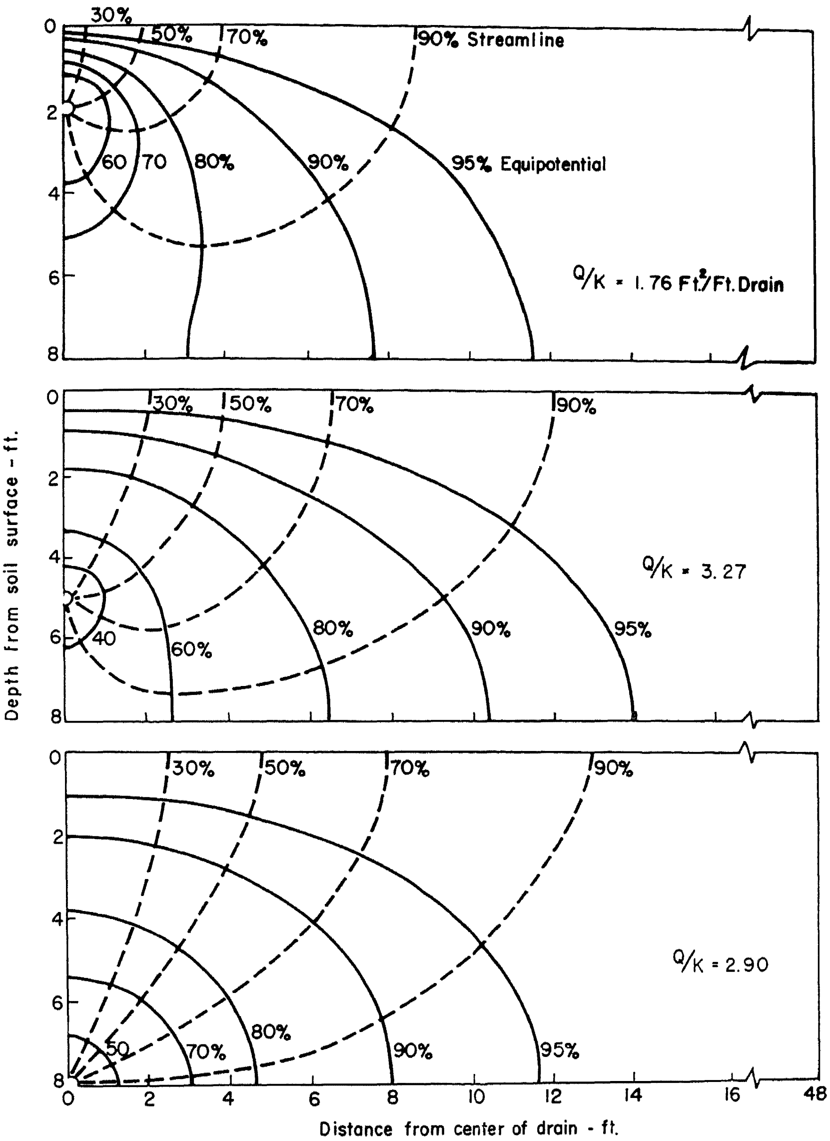

Fig. 7.—Flow nets for three drain depths in the two layered soil illustrated by Case I. The two layers have the same hydraulic conductivity. The 100% equipotential line coincides with the ground surface, while the 0% equipotential is along the drain circumference. The 0% streamline is along the vertical plane passing through and above the drain. The 100% streamline follows the right boundary, the impermeable layer, and the left boundary below the drain.

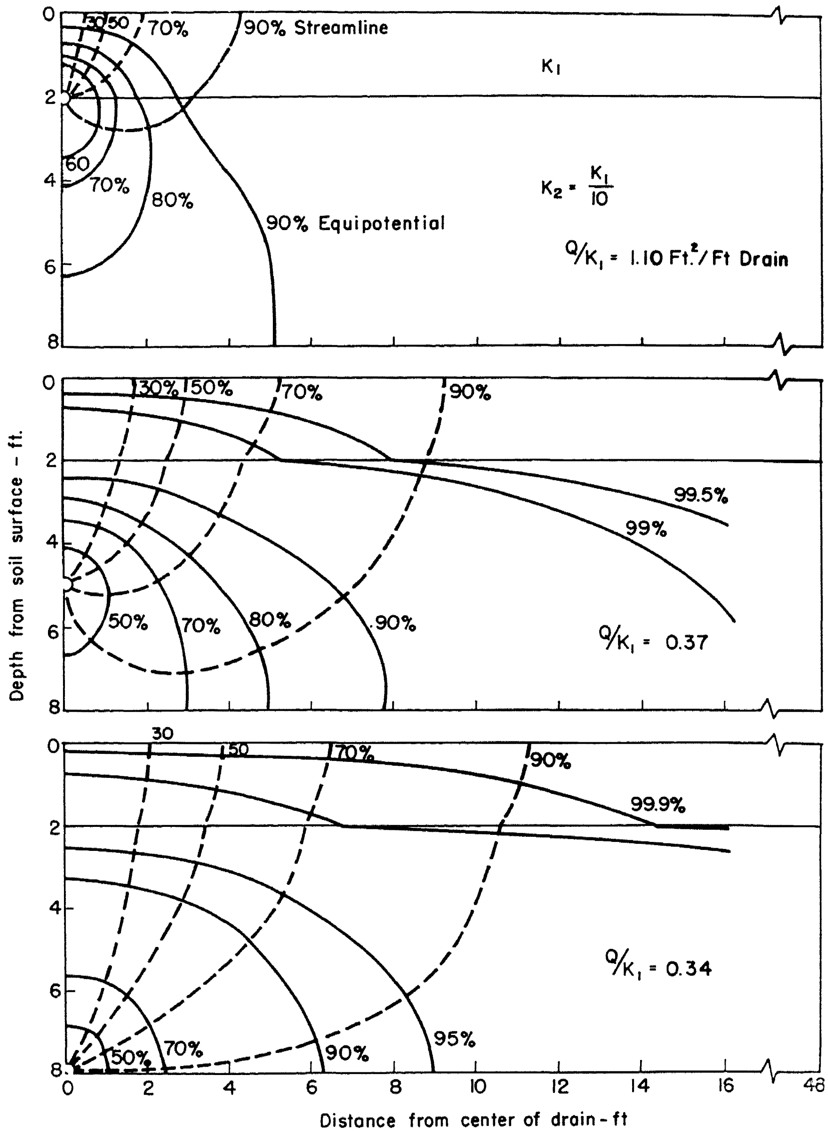

Fig. 8.—Same as figure 7 except that the lower soil layer has a hydraulic conductivity one tenth that of the upper one.

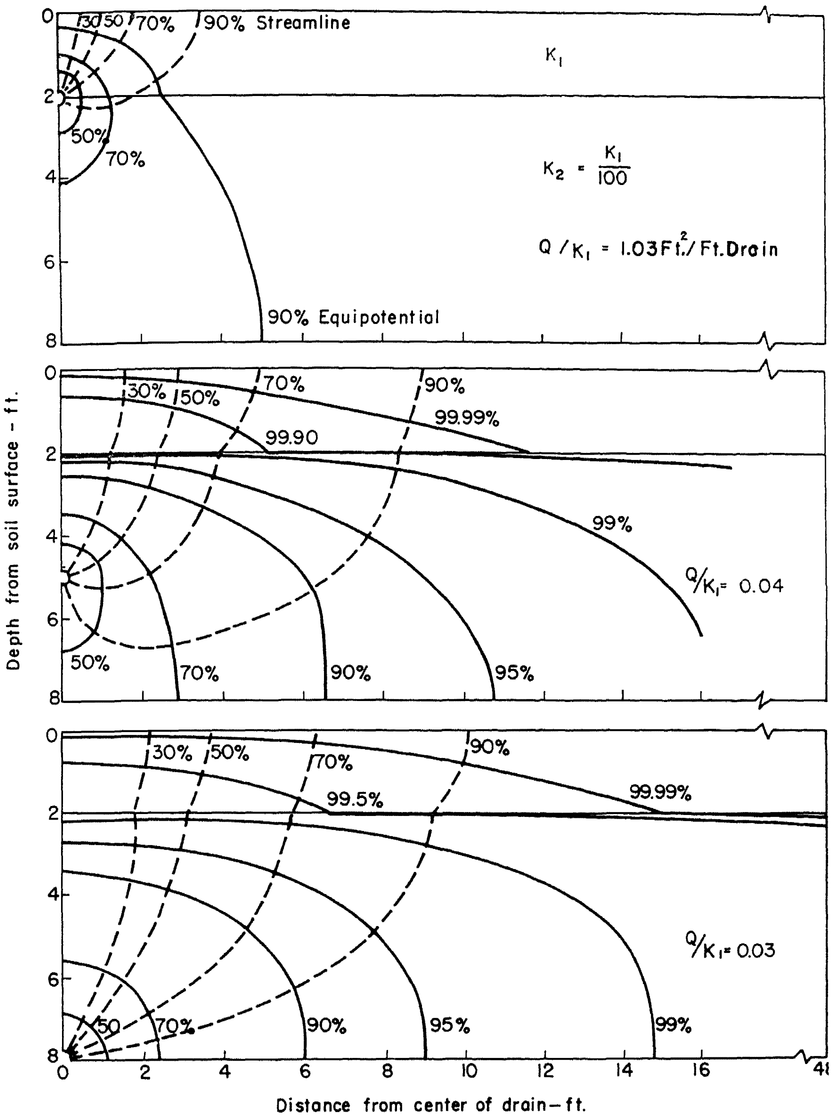

Fig. 9.—Same as figure 7 except that the lower layer has a conductivity one-hundredth that of the upper one.

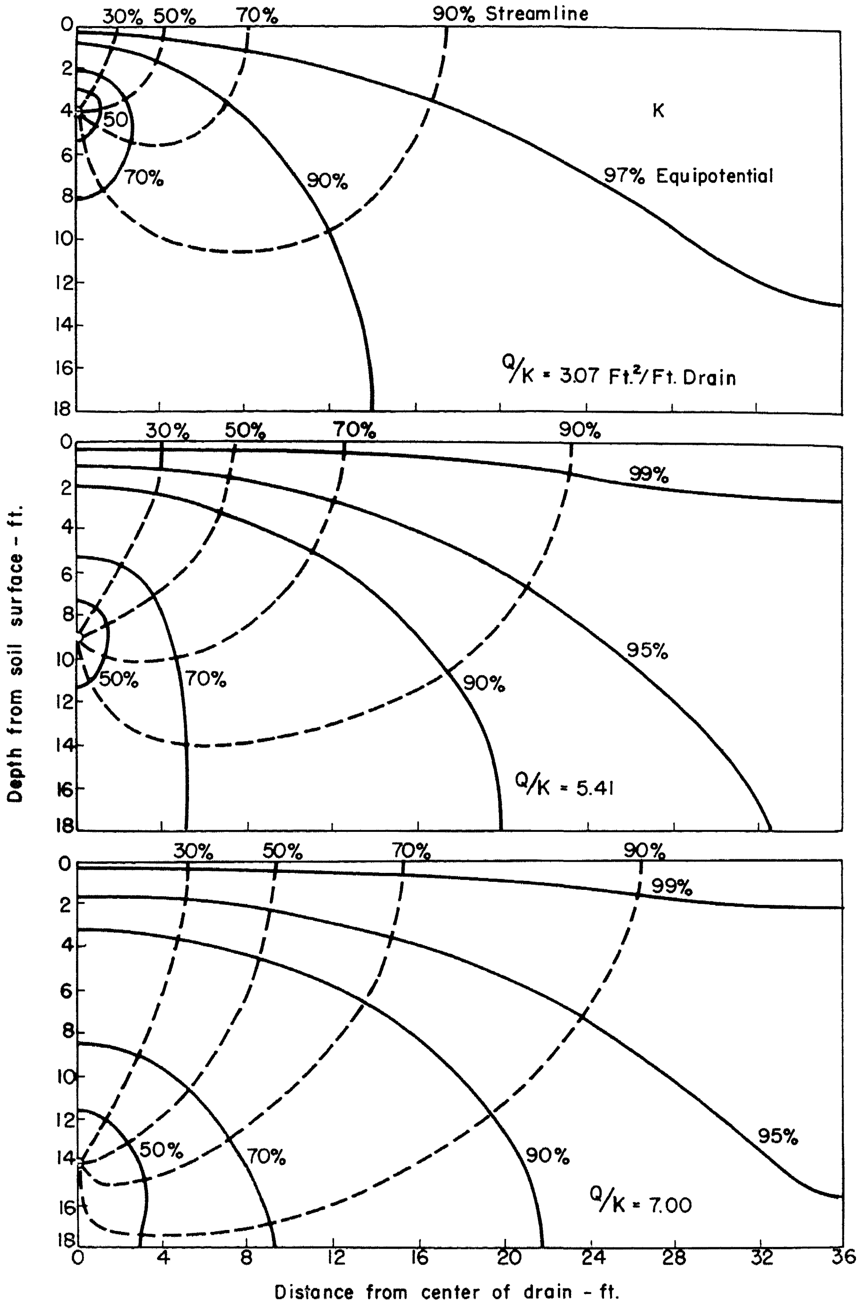

Fig. 10.—Flow nets for three drain depths in the three layered soil illustrated by Case II. All three layers have the same hydraulic conductivity.

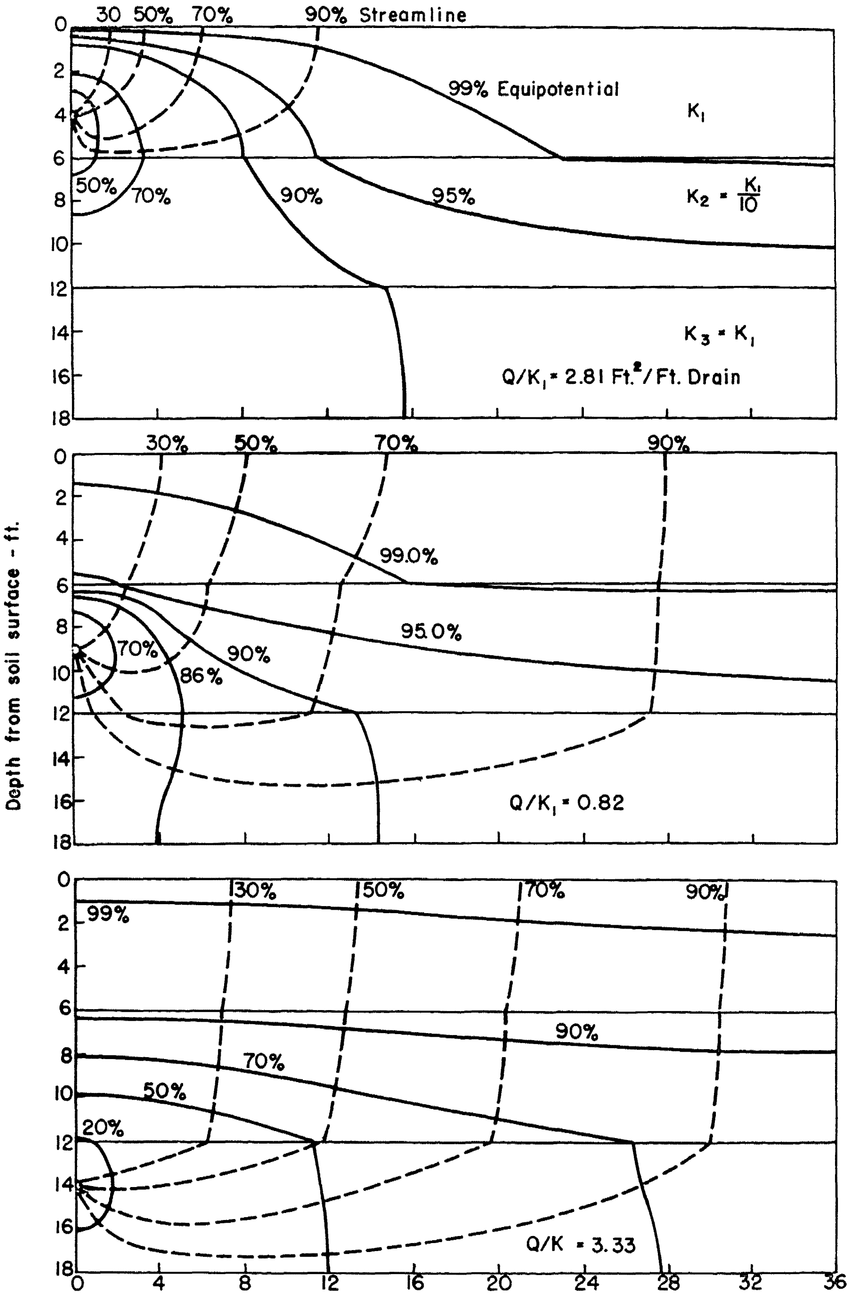

Fig. 11.—Same as figure 10 except that the center soil layer has a conductivity one-tenth that of the other two.

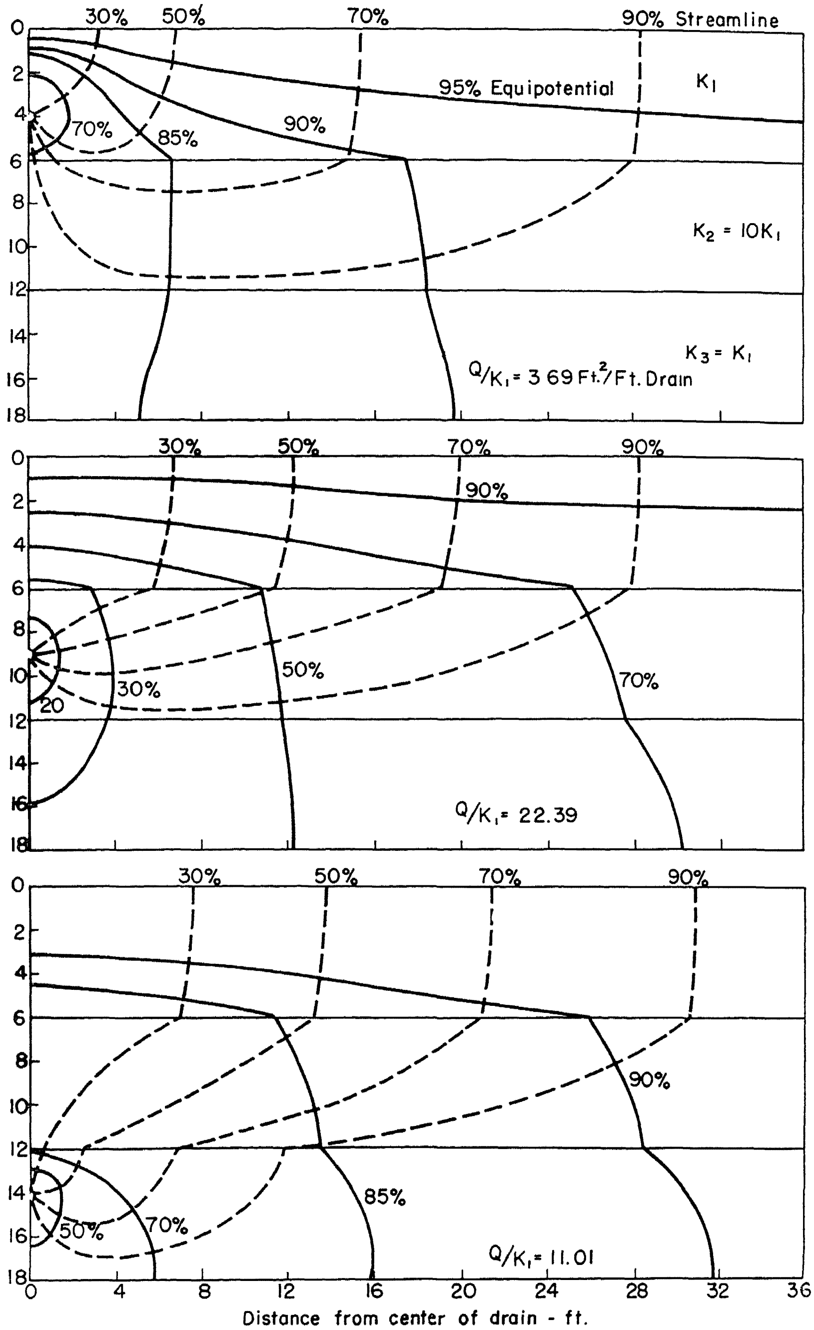

Fig. 12.—Same as figure 10 except that the center soil layer has a conductivity ten times that of the other two.

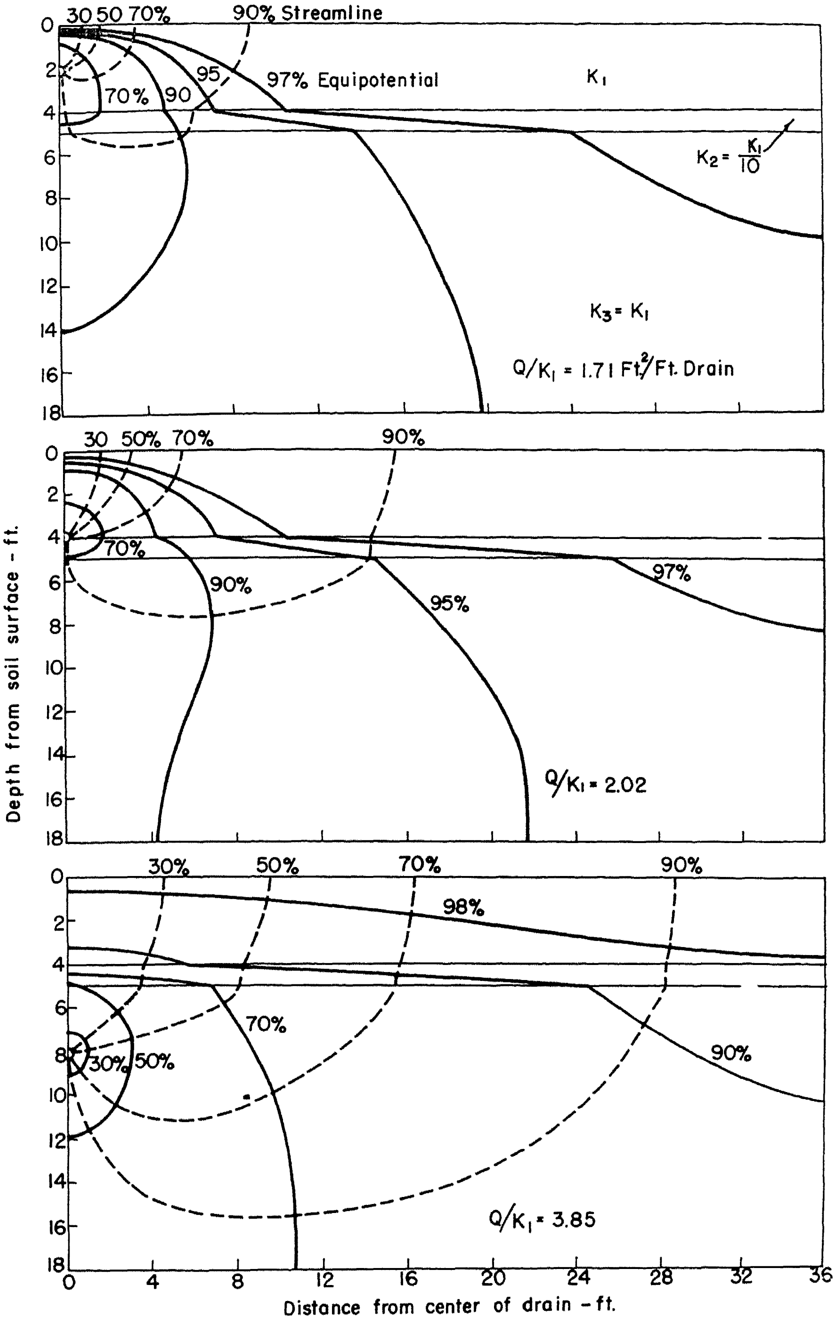

Fig. 13.—Flow nets for three drain depths in the three-layered soil illustrated by Case II. The thin center layer has a conductivity one-tenth that of the other two.